Stochastic Cooling in the Fermilab AntiProton Source

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Wine & Cheese Seminar

2 Feb 2001

From Webster's Collegiate Dictionary

Main Entry: **sto·chas·tic**

Pronunciation: st&-'kas-tik, stO-

Function: *adjective*

Etymology: Greek *stochastikos* skillful in aiming, from *stochazesthai* to aim at, guess at, from *stochos* target, aim, guess -- more at **STING**

Date: 1923

1 : **RANDOM**; *specifically* : involving a random variable <a *stochastic* process>

2 : involving chance or probability : **PROBABILISTIC** <a *stochastic* model of radiation-induced mutation>

- **sto·chas·ti·cal·ly** /-ti-k(&-)lE/ adverb

Main Entry: [2]**cool**

Date: before 12th century

intransitive senses

1 : to become cool : lose heat or warmth <placed the pie in the window to *cool*> -- sometimes used with off or down

2 : to lose ardor or passion <his anger *cooled*>

transitive senses

1 : to make cool : impart a feeling of coolness to <*cooled* the room with a fan> -- often used with *off* or *down* <a swim *cooled* us off a little>

2 a : to moderate the heat, excitement, or force of : CALM <*cooled* her growing anger> b : to slow or lessen the growth or activity of -- usually used with *off* or *down* <wants *to cool off* the economy without freezing it -- Newsweek>

- **cool it** : to calm down : go easy <the word went out to the young to *cool it* -- W. M. Young>

- **cool one's heels** : to wait or be kept waiting for a long time especially from or as if from disdain or discourtesy

Why an Antiproton source?

- p pbar physics with one ring
  - Dense, intense beams for high luminosity
  - Run II Goals
    - 36 bunches of 3 x 10^{10} pbars
    - Small energy spread
    - Small transverse dimensions
  - Collect ~2 x 10^{-5} pbars/proton on target
  - Large Energy Spread & Emittance
    - MANY CYCLES
    - Store and Accumulate
      - COOL!

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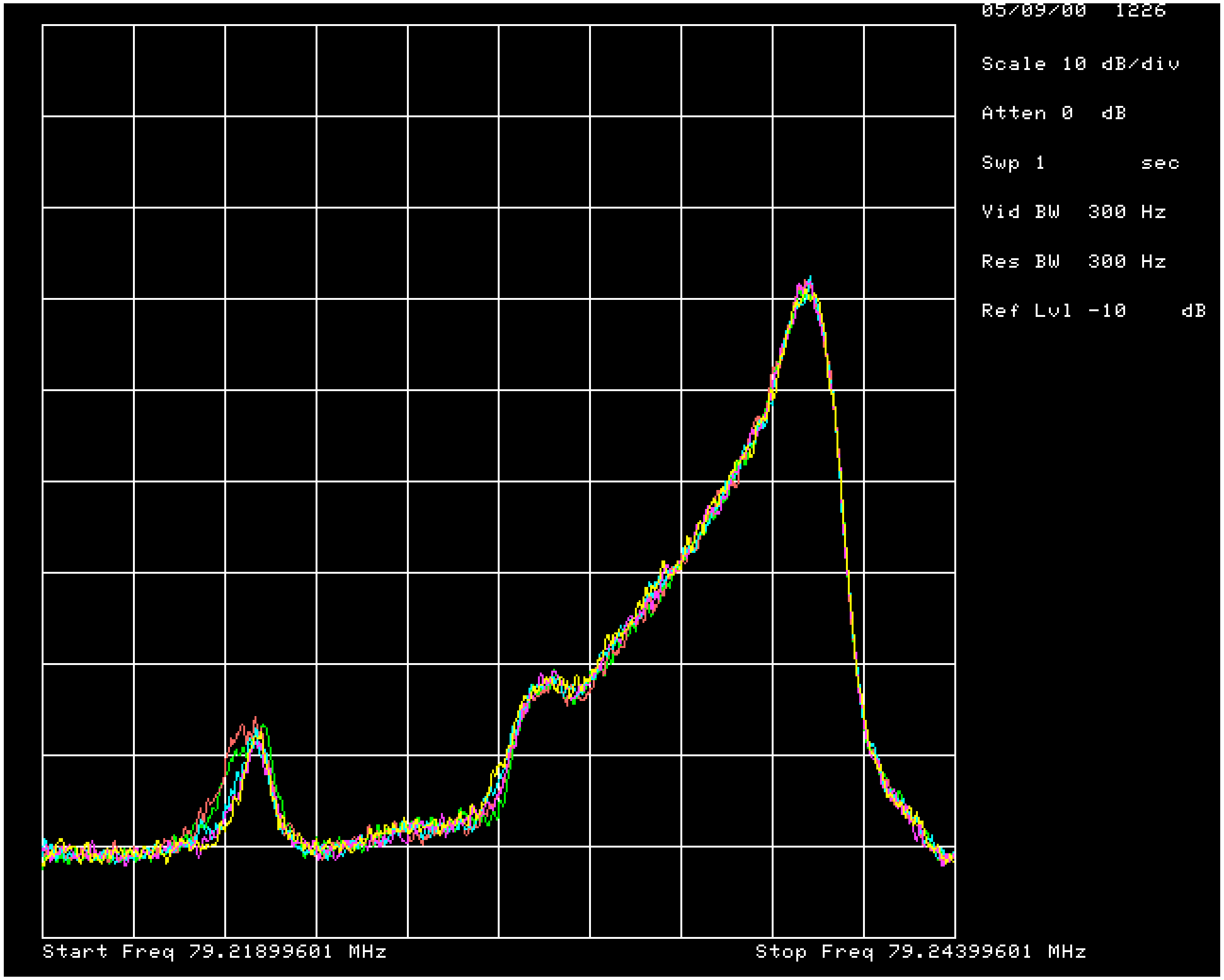
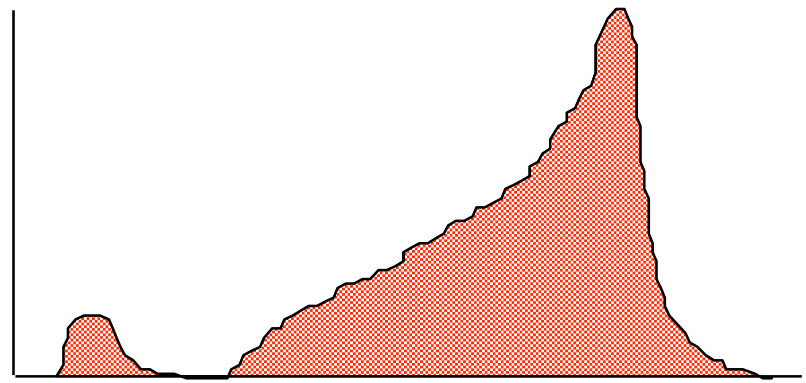
Pbar Longitudinal Distribution



Overview Information

- Frequency Spectrum
  - Time Domain:
    $\sum \delta(t+nT_0)$ at pickup
  - Frequency Domain:
    harmonics of revolution frequency $f_0 = 1/T_0$
- Accumulator:
  T_0~1.6 μsec (1e10 pbar = 1 mA)
  f_0 (core) 628890 Hz
  127th Harmonic ~79 MHz



$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

$$\frac{\Delta f}{f} = -\eta \frac{\Delta p}{p}$$

Idea Behind
Stochastic Cooling

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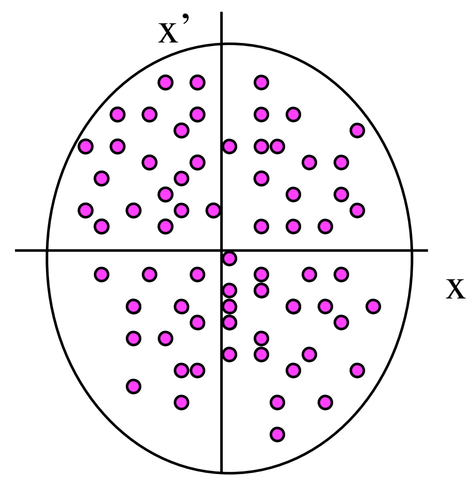
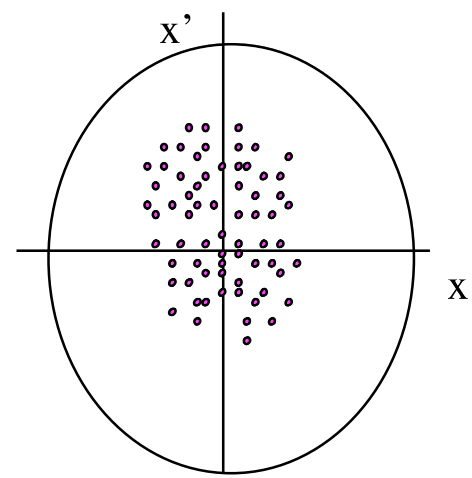
- Phase Space Compression:

  Dynamic Aperture: Area where particles can orbit

  Liouville's Theorem: Local Phase Space Density for conservative system is conserved

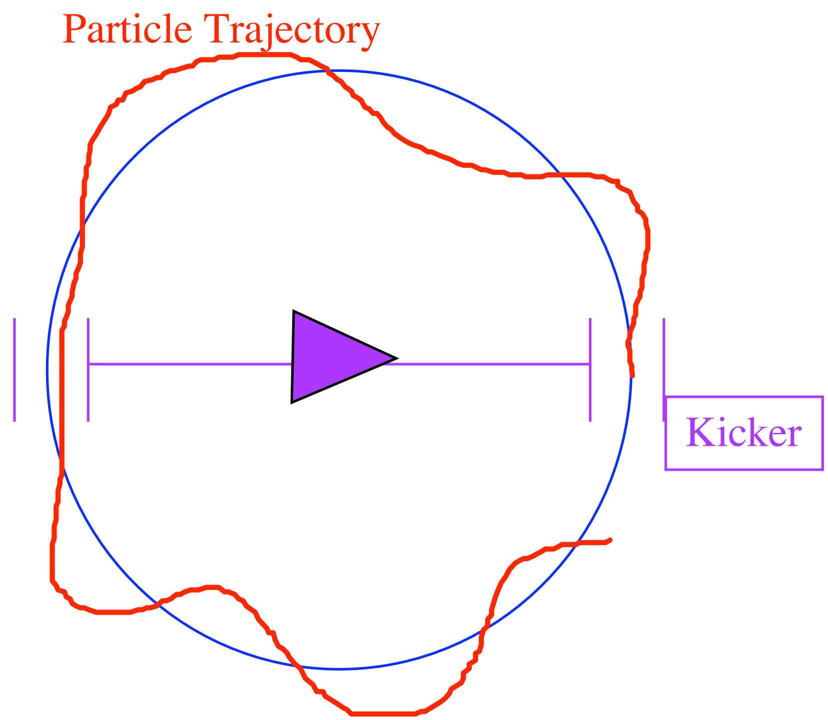
  WANT TO INCREASE PHASE SPACE DENSITY!





Idea Behind
Stochastic Cooling

- Principle of Stochastic cooling
  - Applied to horizontal βtron oscillation
- A little more difficult in practice.
- Used in Debuncher and Accumulator to cool horizontal, vertical, and momentum distributions
- COOLING? Temperature ~ <Kinetic Energy>
  minimize transverse KE
  minimize ΔE longitudinally



Why more difficult in practice?

- Standard Debuncher Operation:
  - 10^8 particles, ~uniformly distributed
  - Central revolution frequency 590035 Hz
    - » Resolve 10^{-14} seconds to see individual particles!
    - » 100 THz antennas $\lambda = 3\ \mu m$!
  - pickups, kickers, electronics in this frequency range ?
  - Sample N_s particles -> Stochastic process
    - » $N_s = {}^{N}/_{2TW}$ where T is revolution time and W bandwidth
    - » Measure <x> deviations for N_s particles
  - Higher bandwidth the better the cooling

Simple Betatron Cooling

With correction ~ **g<x>**, where g is related to gain of system

- New position: x - **g<x>**

- Emittance Reduction: RMS of kth particle

$$(x_k - g\langle x\rangle)^2 = x_k^2 - 2g x_k + g^2 \langle x\rangle^2$$

$$\langle x\rangle = \frac{1}{N_s}\sum_i x_i = \frac{1}{N_s} x_k + \frac{1}{N_s}\sum_{i \neq k} x_i$$

Average over all particles and do lots of algebra

$$\frac{d\langle x\rangle^2}{dn} = \frac{-2g\langle x^2\rangle}{N_s} + \frac{g^2}{N_s}\langle x^2\rangle, \text{ where n is 'sample'}$$

$$\Rightarrow \text{ Cooling Time } \frac{1}{\tau} = \frac{2W}{N}\left(2g - g^2\right)$$

Stochastic Nature?

- Result depends upon independence of the measured centroid <x> in each sample
  - In case where have no frequency spread in beam, cannot cool with this technique!

$$\frac{\Delta f}{f} = -\eta \frac{\Delta p}{p}$$

  - Some number of turns **M** to completely generate independent sample
- But…
  - Where is randomization occurring?
    - » WANT: kicker to pickup GOOD MIXING
    - » ALSO HAVE: pickup to kicker BAD MIXING

Cooling Time

- Electronic Noise:
  - Random correction applied to each sample
  - More likely to heat than cool
  - Noise/Signal Ratio **U**

$$\Rightarrow \text{ Cooling Time } \frac{1}{\tau} = \frac{2W}{N}\left(2g - g^{2}[M + U]\right)$$

  - High Bandwidth
  - Low Noise

  - Optimum Gain (in correction **g**) goes down as N goes up!

Momentum Cooling

- Time evolution of the particle density function, $\psi(E) = {}^{\partial N}/_{\partial E}$
  - Fokker-Planck Equation -- c. 1914
    first used to describe Brownian motion

$$\frac{\partial \psi}{\partial t} = -\frac{\partial}{\partial E}\left[F(E)\psi - D(E)\frac{\partial \psi}{\partial E}\right]$$

$F(E)$ is gain function

$D(E)$ represent diffusion terms (noise, mixing, feedback)

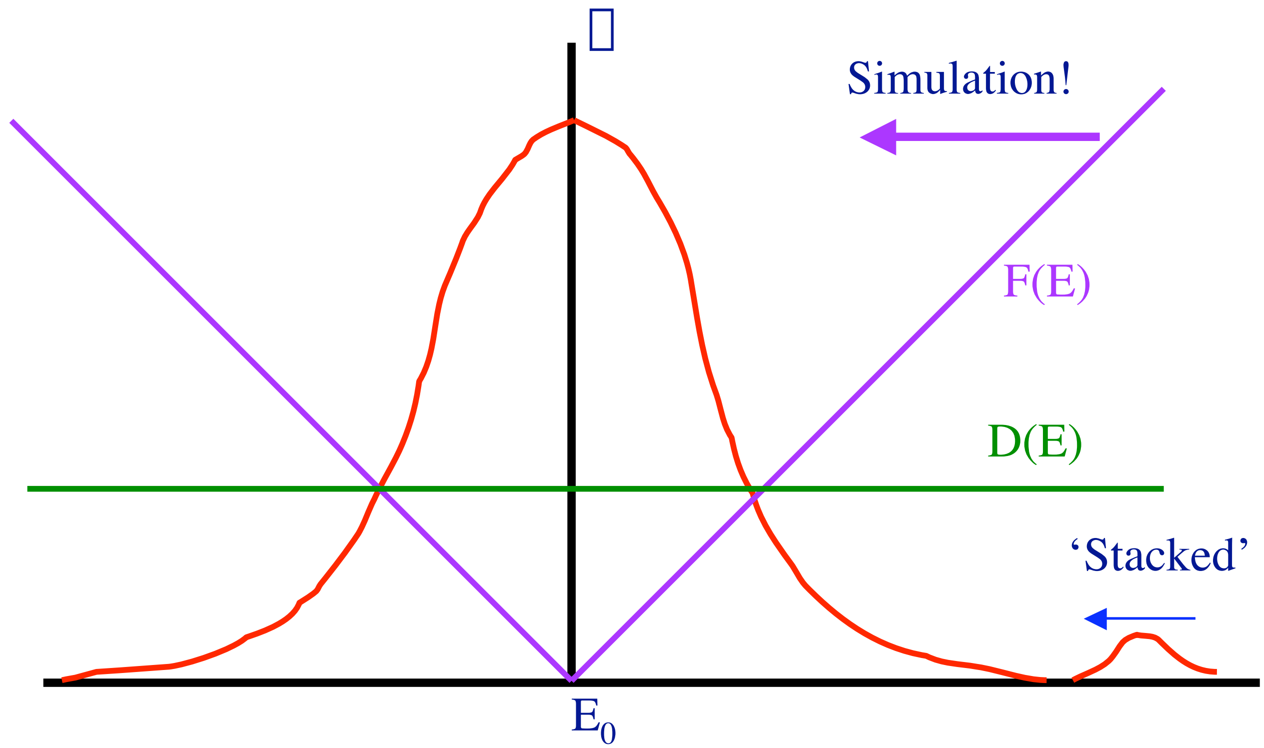
- Two Pieces:
  - Coherent self force through pickup, amplifier, kicker
    - » Directed motion of the particle
  - Random kicks from other particles and electronic noise
    - » Diffusive flux from high density to low density

Simple Example

$$F(E) = -\lambda(E - E_0)$$

$$D(E) = D_0$$

$$\Rightarrow \quad \psi(E) = \psi_0 e^{-\frac{\lambda}{2D}(E - E_0)^2}$$



- Linear Restoring Force with Constant Diffusive Term (Electronic noise)
  - Gaussian Distribution
- Inject at E> E_0
  - Coherent force dominates --- collected into core!

Momentum Stacking

Van der Meer's solution: desire constant flux past energy point

$\Rightarrow$ static solution !

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial E}\left(F(E)\psi(E) - D(E)\frac{\partial \psi}{\partial E}\right) = \frac{\partial}{\partial E}\Phi = 0$$

$V(E)$ volts per turn applied at kicker

diffusive term depends upon particle density and mean square voltage applied

(ignoring amplifier noise for the moment)

Φ_0 constant flux

$F(E) = eV/T$ where T is period

$$D(E) = AV(E)^2\psi(E)$$

$$\Rightarrow \Phi_0 = \frac{eV}{T}\psi - \psi AV^2\frac{\partial \psi}{\partial E}$$

Van der Meer's Solution

$$\frac{\partial \square}{\partial E} = \square \frac{\square_0}{AV^2 \square} + \frac{e}{AVT}$$ & Maximize Gradient term

$$V = \frac{2\square_0 T}{e\square}$$

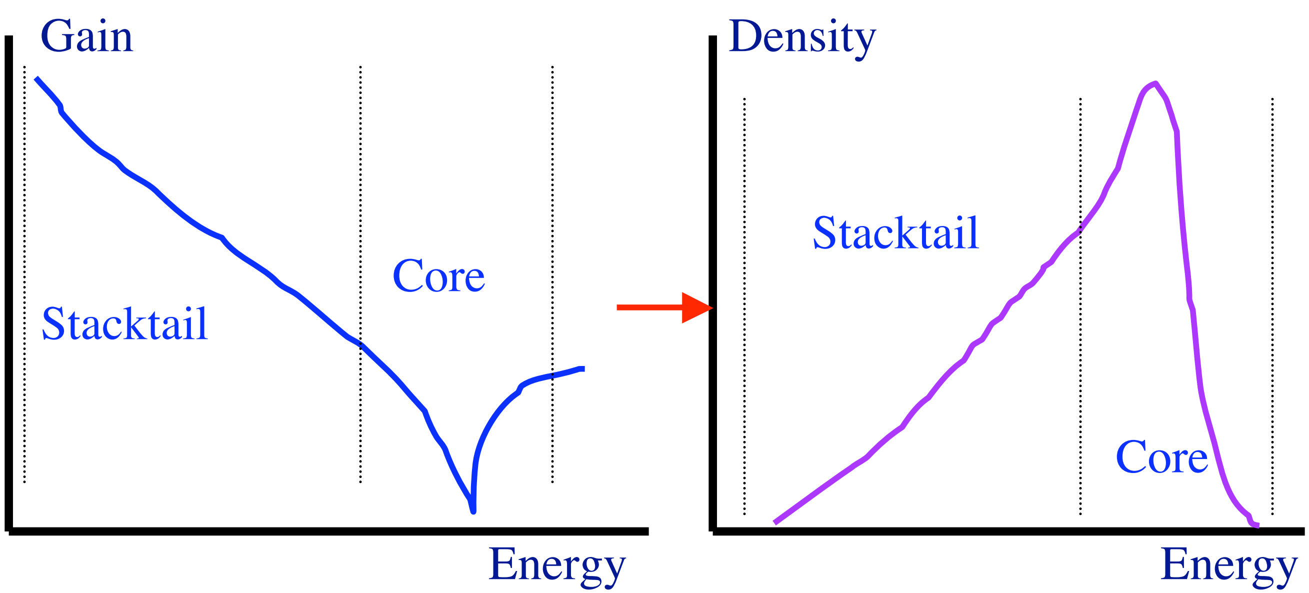
Substitute and Integrate

$$\square\ \ \square = \square_0 e^{(E \square E_i)/E_d}$$, where $E_d = 4A\square_0 T^2 / e^2$

$$V = \frac{2\square_0 T}{e\square_0} e^{\square(E \square E_i)/E_d}$$

To build constant flux, build voltage profile which is exponential in shape and results in density distribution which is exponential in shape!

- Exponential Density Distribution generated by Exponential Gain Distribution
- Max Flux = $(W^2|\square|E_d)/(f_0 p \ln(2))$



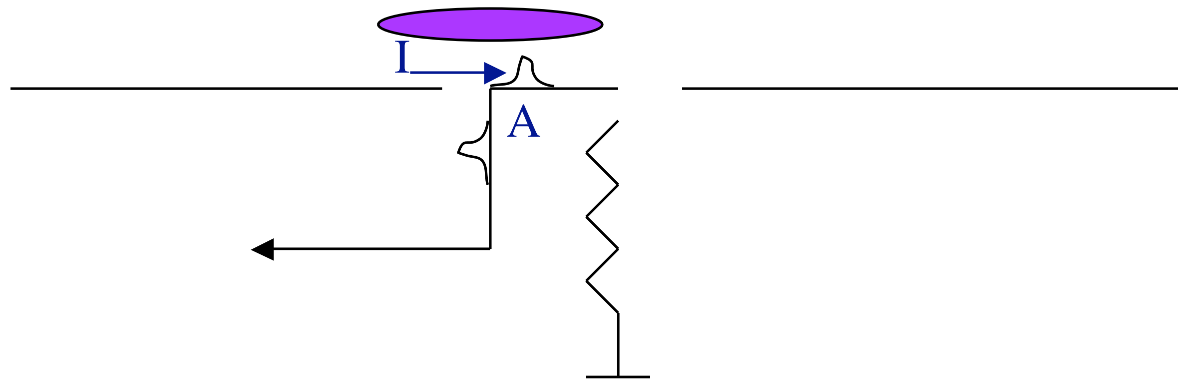
Using log scales on vertical axis

Implementation in Accumulator

- How do we build an exponential gain distribution?
- Beam Pickups:
  - Charged Particles: E & B fields generate image currents in beam pipe
  - Pickup disrupts image currents, inducing a voltage signal
  - Octave Bandwidth (1-2, 2-4,4-8 GHz)
  - Output is combined using binary combiner boards to make a phased antenna array

Beam Pickups

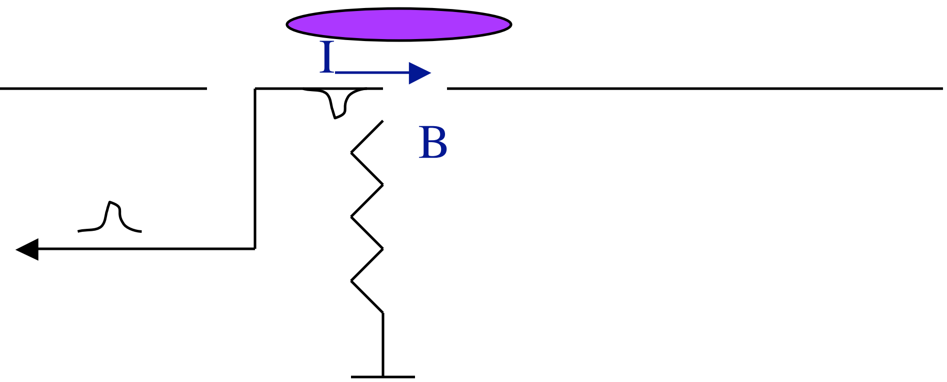
- At A:



Current induced by voltage across junction splits in two, 1/2 goes out, 1/2 travels with image current

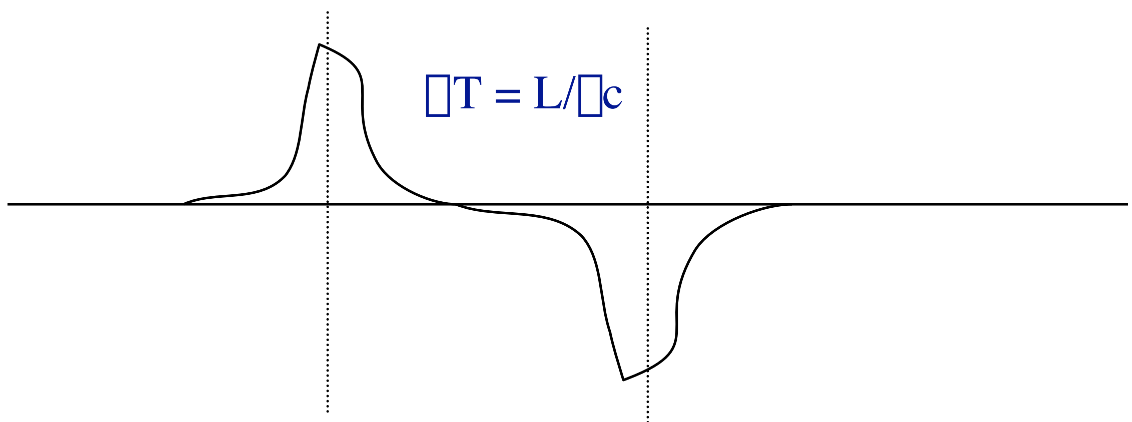
Beam Pickups

- At B:



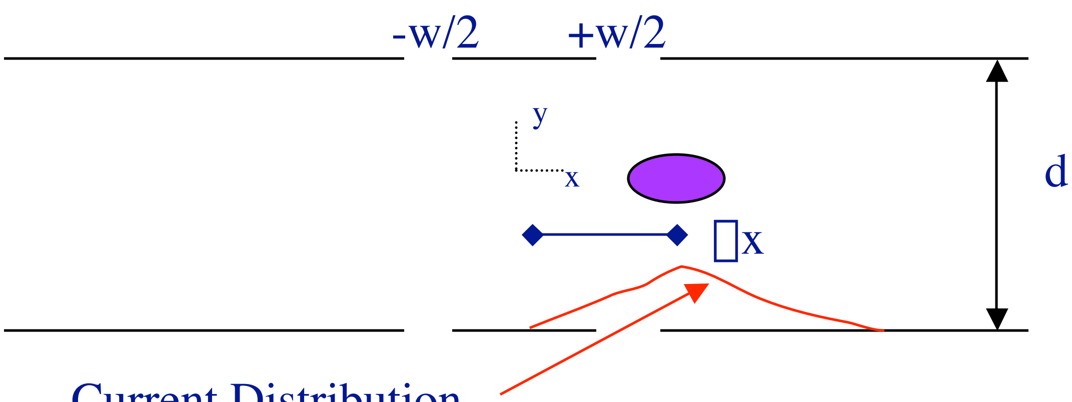
Current splits in two paths, now with OPPOSITE sign

- Into load resistor ~ 0 current
- Two current pulses out signal line



Current Intercepted by Pickup

Use Method of Images



$$I = \frac{I_{beam}}{\pi}\left[\tan^{-1}\left(\sinh\left(\frac{\pi}{d}\right)\left(x + \frac{w}{2}\right)\right) - \tan^{-1}\left(\sinh\left(\frac{\pi}{d}\right)\left(x - \frac{w}{2}\right)\right)\right]$$

$$\approx \frac{I_{beam}}{\pi}\exp\left(-\frac{\pi \Delta x}{d}\right) \text{ for large } \Delta x \; (\Delta x \geq w)$$

- In areas of momentum dispersion D

$$\Delta x = \frac{D}{\beta^2}\frac{\Delta E}{E}$$

- Placement of pickups to give proper gain distribution

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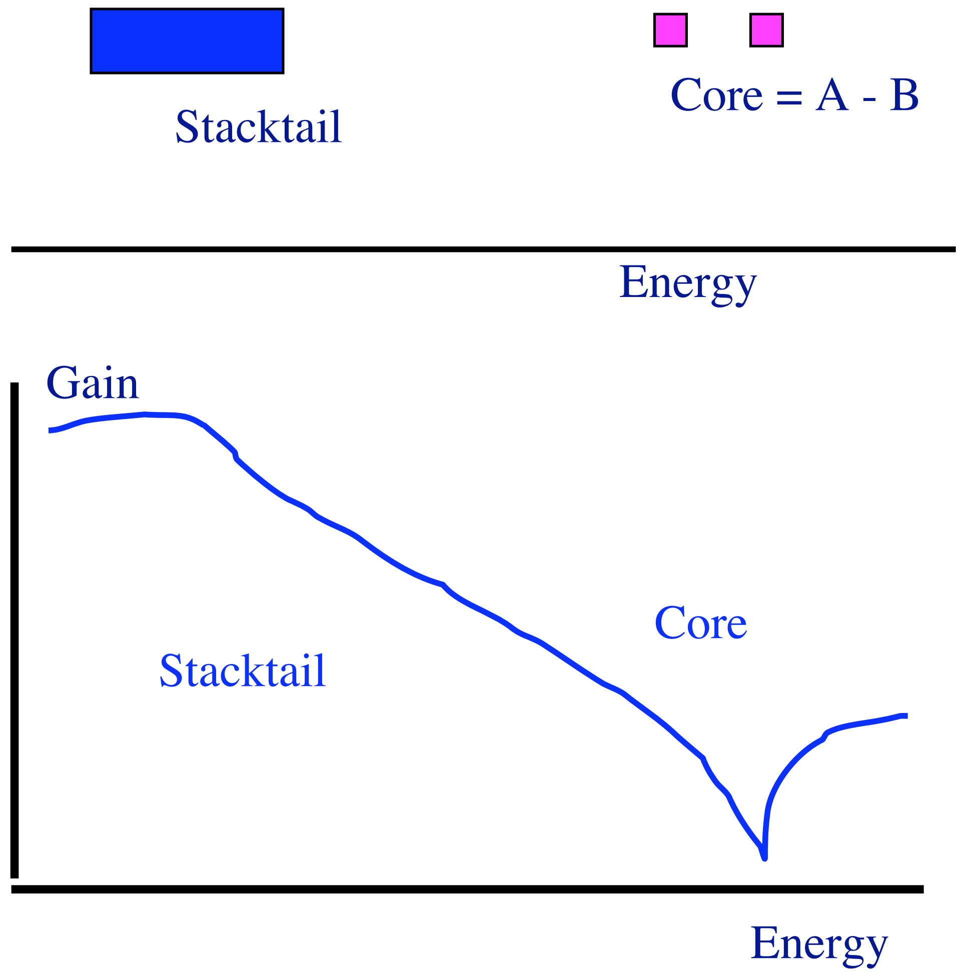
Accumulator Pickups

- Placement
- number of pickups
- amplification
- used to build gain shape

- Also use Notch filters to zero signal at core



Accumulator Stacktail

- Not quite as simple:
  - -Real part of gain cools beam
  - frequency depends on momentum
    $\Delta f/f = -\eta\Delta p/p$ (higher f at lower p)
  - Position depends on momentum
    $\Delta x = D\Delta p/p$
  - Particles at different positions have different flight times
  - Cooling system delay constant
    - » **OUT OF PHASE WITH COOLING SYSTEM AS MOMENTUM CHANGES**

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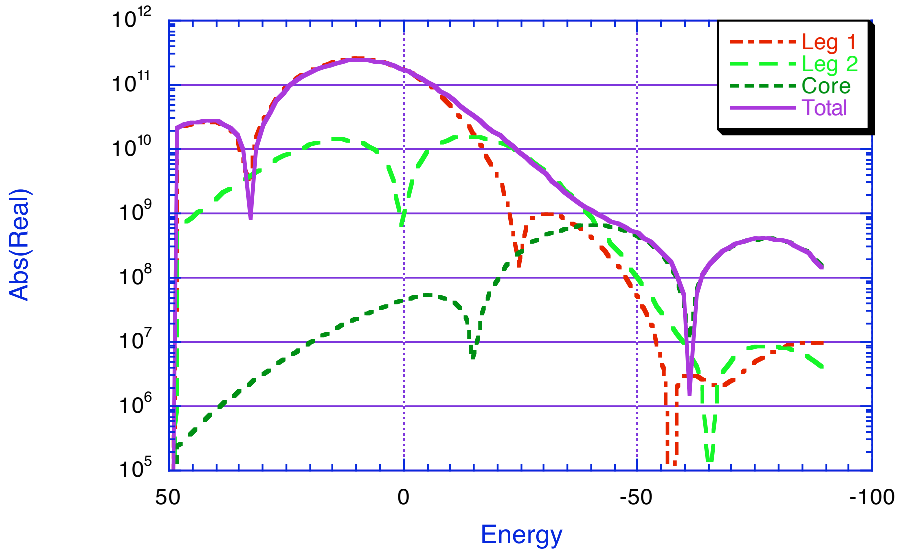
Accumulator Stacktail



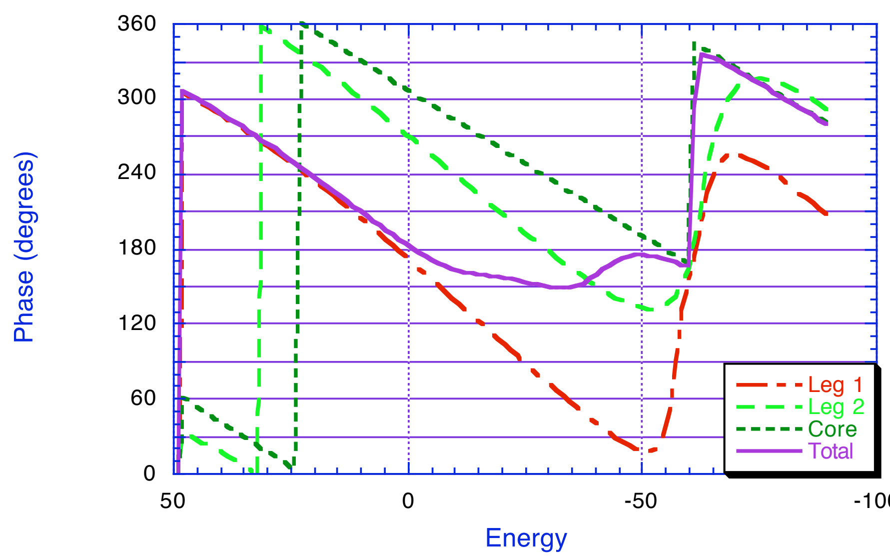
Use two sets of pickups at different Energies to create exponential Distribution with desired phase Characteristics

Stacktail Design Goal For Run II

$E_d \sim 7$ MeV

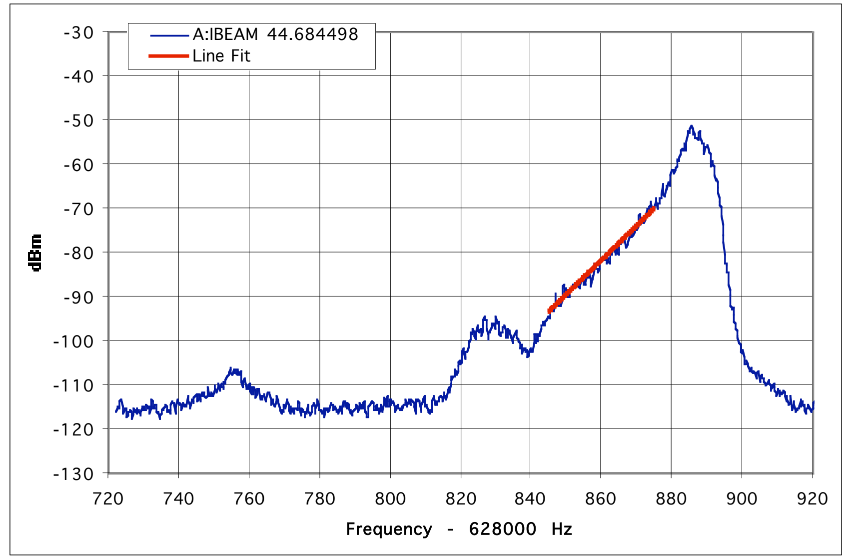
Flux ~ 35 mA/hour

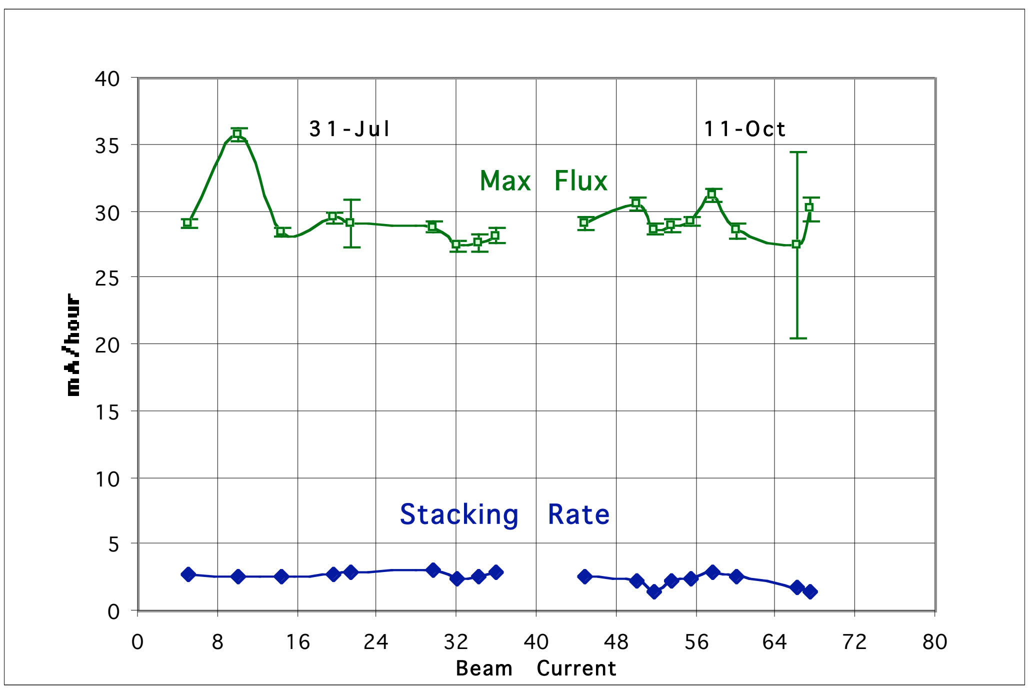
Show simulation!



Performance Measurements

- Fit to exponential in region of stacktail (845-875 in these units)
- Calculate Maximum Flux for fitted gain shape
- Different beam currents

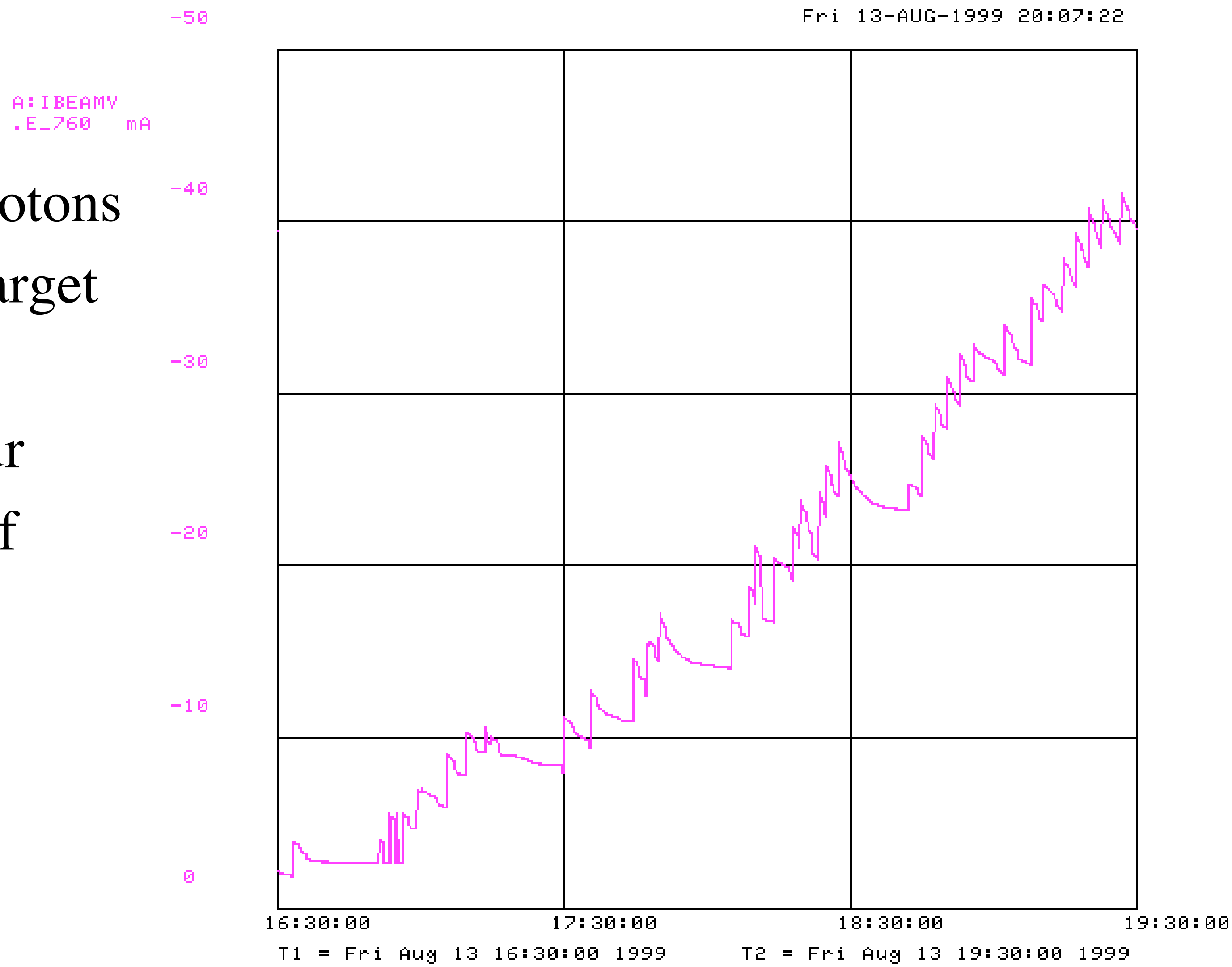




- Independent of Stack Size
- Max Flux ~30 mA/hour

Performance Measurements

- Inject 8 GeV protons
  - Bypassing target
- ‘Stacking rate’
  - ~13 mA/hour
  - First week of operation!



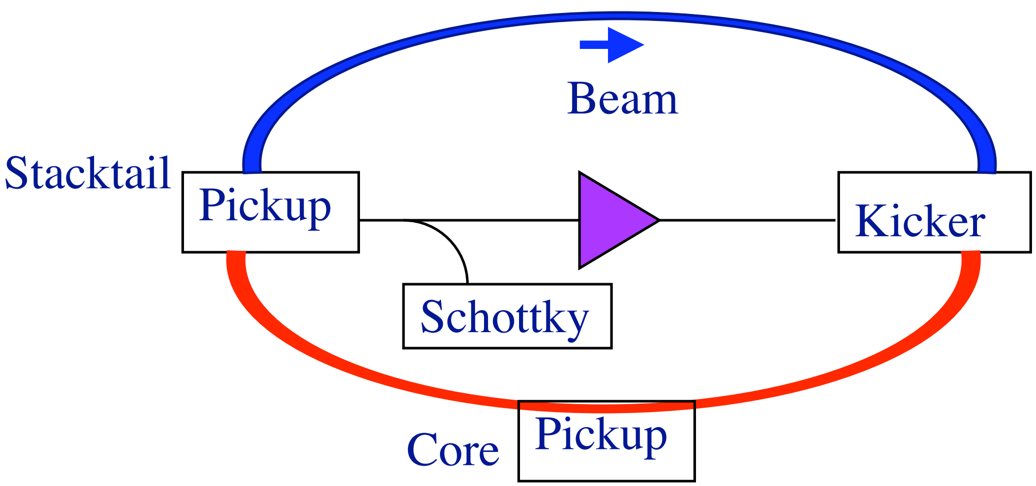
Performance Measurements

	Engineering Run	Run IIa Goal
Protons on Target	3.8e12	5e12
Cycle Time	3.2	1.5
Production Efficiency (pbars/10^6protons)	10	20
Stacking Rate (1e10 per hour)	4	18

- Stacking rate limited by input flux and cycle time

Stacktail - Core Coupling

- Coupling in regions where frequency bands overlap
  - 2-4 GHz ! much larger than previous overlap
- Two phenomena
  - Coherent beam feedback
    - » Stacktail kicks beam and coherent motion is seen at core
  - Misalignment gives transverse - longitudinal coupling
    - » Try to correct with kickers



Since beam does not decohere,
Carry information back to pickup

Feedback!

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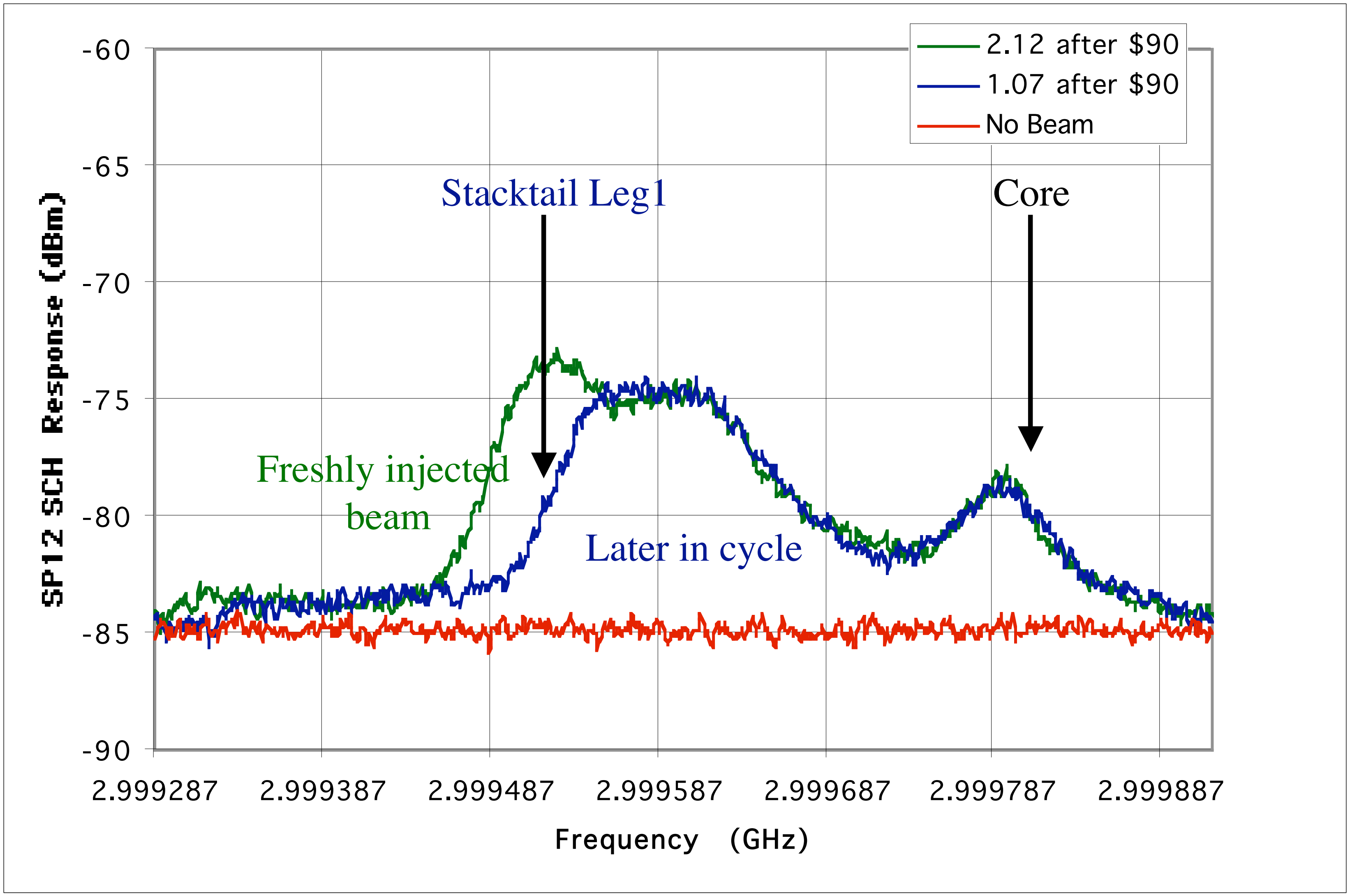
Stacktail Schottky Signals



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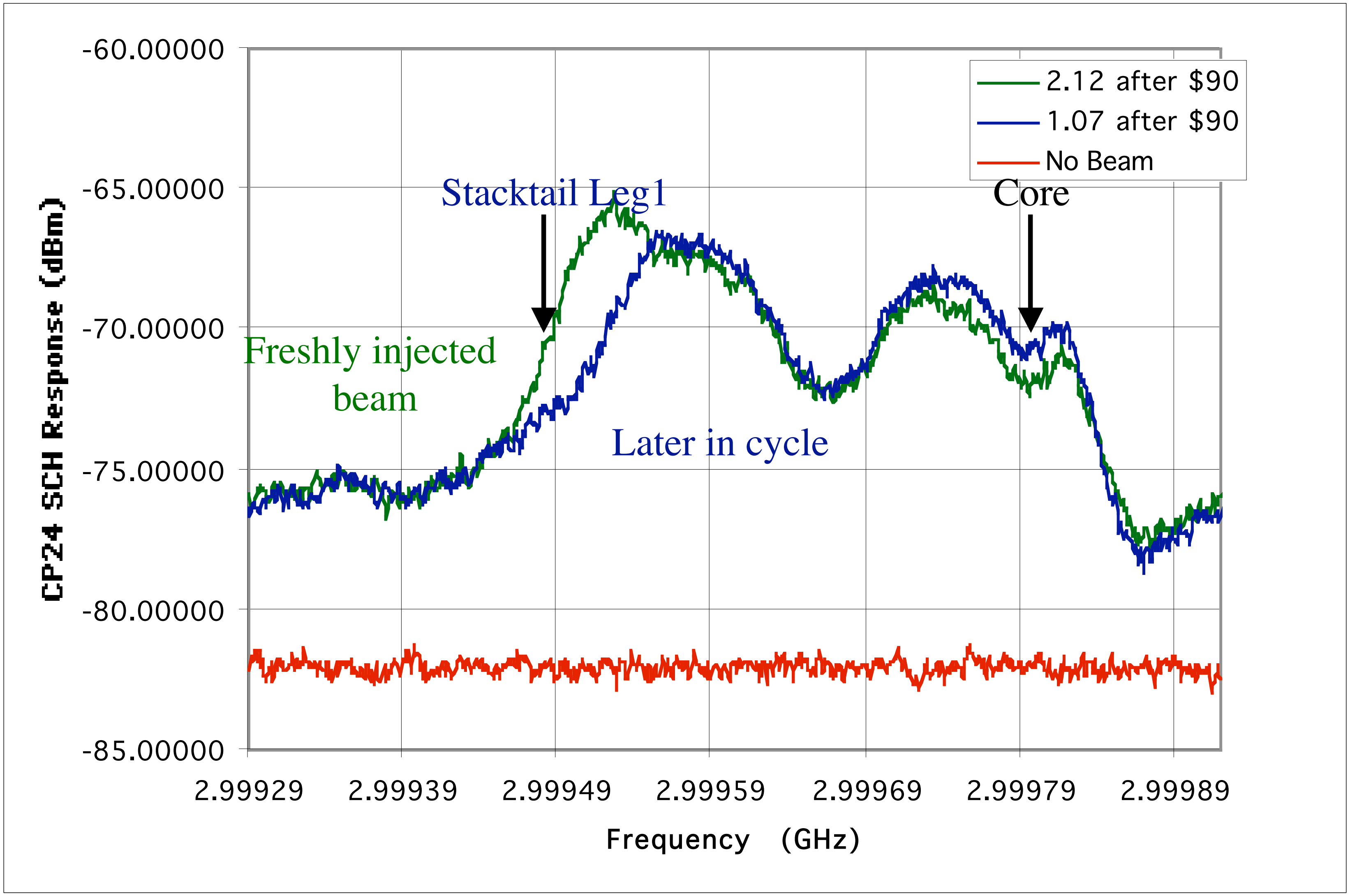
Core 2-4 Schottky Signals



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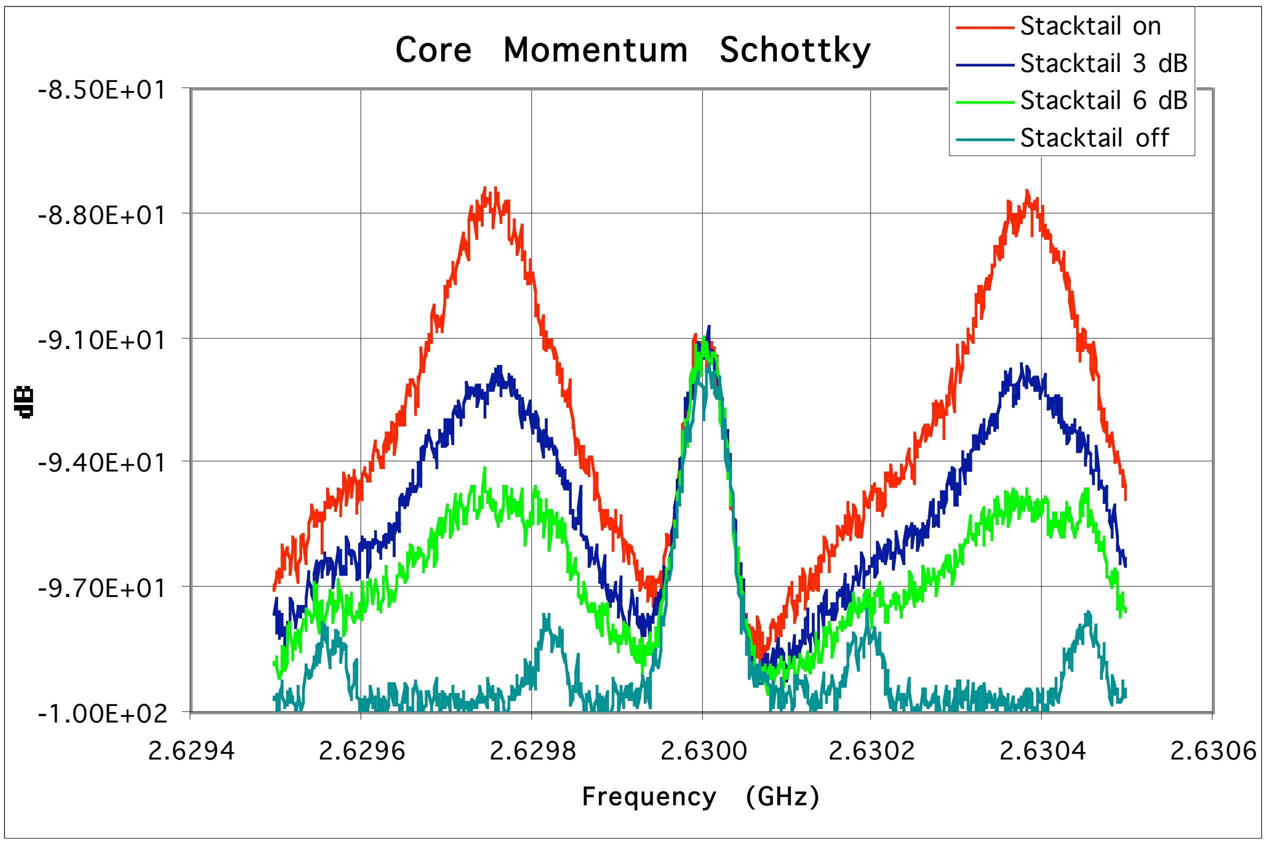
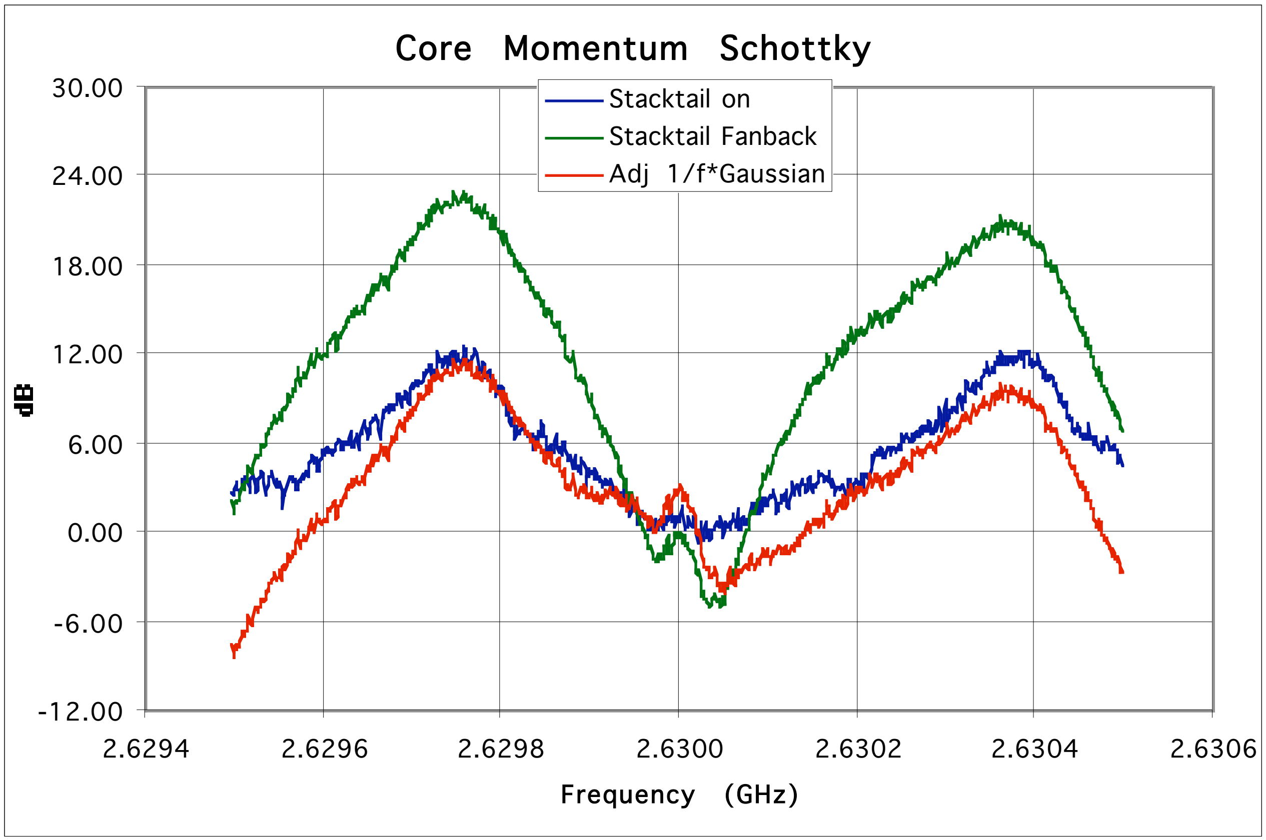
Core 2-4 Schottky Signals

- No injected beam
- Stacktail ON!



Coherent Beam Feedback

- Use Stacktail FB spectrum
- Simple model of feedback effects
  - Reasonable explanation of shape



Pbar Longitudinal Distribution

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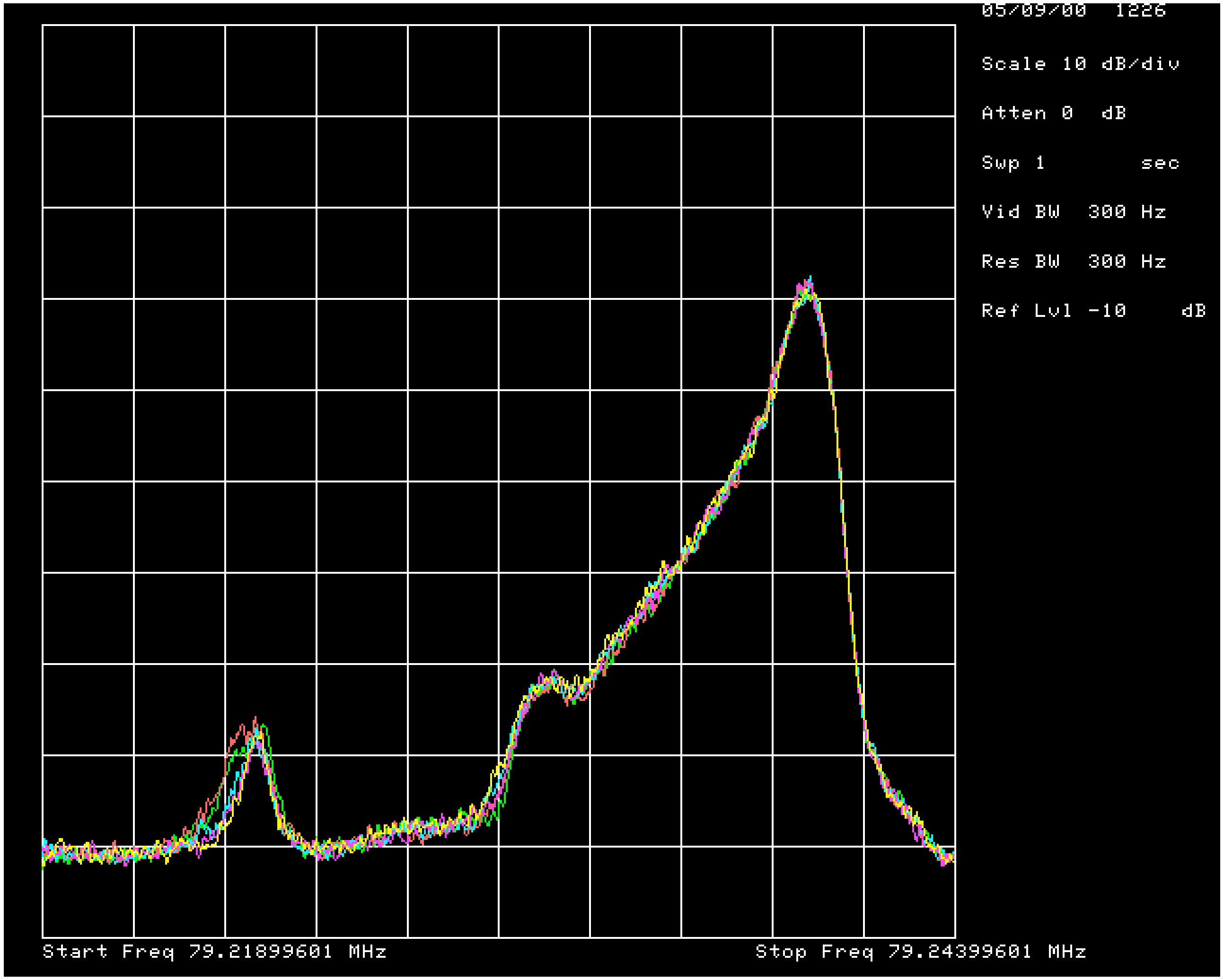

Antiprotons & the Collider

- From the H$^-$ source, Linac, booster, Main Injector
  - 120 GeV protons on the target
- From the target:
  - 8 GeV antiprotons through the Debuncher & Accumulator
- Send them off to the Tevatron & D0 & CDF